THE POWER-CONJUGACY PROBLEM FOR THE HIGMAN-THOMPSON GROUP $G_{n,r}$

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ABSTRACT. The Higman-Thompson group $G_{n,r}$ $(n \geq 2, r \geq 1)$ was defined in 1974 as the automorphism group of a certain free algebra. We use the solution to the conjugacy problem given by Graham Higman to provide a solution to the power conjugacy problem for the family of groups $G_{n,r}$ $(n \geq 2, r \geq 1)$, of which Richard Thompson's group V $(n = 2, r = 1)$ is a member.

1. INTRODUCTION

The conjugacy problem is a classic decision problem first outlined by Max Dehn over 100 years ago. It asks, for a group G, "does there exist an algorithm to decide whether, upon input of the elements $\psi, \varphi \in G$ there exists an element $g \in G$ such that $g^{-1}\psi g = \varphi$?"

Once we know that the answer to this question is "yes" then one can ask other conjugacy based decision problems about the group G. We describe some motivation for the following question in Section 1.1.

Question 1.1. [1] Does there exist $a, b \in \mathbb{Z}$ and $z \in G$ such that $x^a = z^{-1}y^b z \neq 1$ for $x, y \in G$?

In 1974 Graham Higman [3] constructed an infinite family of groups $G_{n,r}$ for $n \geq 2$ and $r \in \mathbb{N}$ such that each group $G_{n,r}$ was a finitely presented infinite group and was either simple for n even or contained a normal subgroup $G'_{n,r}$ which was simple for n odd.

The construction used by Graham Higman was based on a report of F. Galvin and R. Thompson [unpublished] on the work of B. Jónsson and A. Tarski [4]. Thompson observed that the automorphism group of the Jónsson-Tarski algebra of type $\langle 2, 1, 1\rangle$ was isomorphic to Thompson's group V, that is the group $G_{2,1}$ in Higman's notation.

The Higman-Thompson group $G_{n,r}$ (defined in Section 1.4) has been shown to have solvable conjugacy problem [3] and in the current note we show that the power conjugacy problem is solvable for each group $G_{n,r}$.

Theorem 1.2. *The power conjugacy problem for the Higman-Thompson group $G_{n,r}$ $(n \geq 2, r \geq 1)$ is solvable.*

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Remark 1.3. In this note we have use the techniques and tools of Graham Higman [3], which involves universal algebra. Solutions of the conjugacy problem for the group $G_{2,1}$, also known as Thompson's group V, are given in [9] using the dynamics of tree pair representatives and in [2] using the work on strand diagrams.

1.1. **Motivation for the power conjugacy problem.** The power conjugacy problem naturally arises when you have any group B and G an HNN group given by

$$\langle a, B | \text{rel } B, a^{-1}Wa = U \rangle,$$

where W and U are words in the generators of B defining elements of the same order. It follows from [7, Lemma 5] that if x and y are elements in B which are conjugate in G but not in B then x and y are conjugate in B to powers of W or U and hence in G to powers of W. This gives rise to the definition of the power conjugate problem for a group G. See [1, 6, 8] for references to this problem.

1.2. **Universal algebras.** We will now describe some useful background material on universal algebra, which can be found in [5, Chapter 1, §1.3] .

Given a set S, we say a mapping $f\colon S^n \to S$ is an n-ary algebraic operation on S.

Definition 1.4. We define an *algebraic system* $\mathcal{A} = (S, f_i : i \in I)$ to consist of a set S together with a set of mappings $\{f_i : i \in I\}$ where each f_i is an n_i-ary algebraic operation on S for some $n_i \in \mathbb{N}$. We call $\langle n_i : i \in I \rangle$ the signature of $\mathcal{A}$.

Definition 1.5. A variety $\mathcal{V}$ is the class of all algebraic systems of a fixed signature defined by a given set of laws that are expressions in the operations of the algebraic systems.

Example 1.6. The class of all groups is the variety defined by the group laws.

An algebraic system $\mathcal{A}$ is free in the variety $\mathcal{V}$ on the set of free generators X if for any element y_j, $j \in J$, in any algebraic system $\mathcal{B} \in \mathcal{V}$, generated by Y, the mappings $x_j \mapsto y_j$ extends to a unique homomorphism of $\mathcal{A}$ into $\mathcal{B}$. A free system $\mathcal{A}$ can be constructed as the set of all formal expressions in the x_j's under arbitrarily repeated applications of the n_i-ary operations f_i.

1.3. **The variety $\mathcal{V}_n$ and the free algebra $V_{n,r}$.** We can formulate in full generality the free algebra $V_{n,r}$ (as Higman did in [3, Section 2]) in terms of universal algebra for an algebraic system consisting of a set S and a one-to-one mapping from S to its n^{th} cartesian product S^n.

The mapping $S \to S^n$ may be described using n mappings $\boldsymbol{\alpha_i} : S \to S$ for $i = 1, ..., n$, where

$$a \mapsto (a\boldsymbol{\alpha_1}, ..., a\boldsymbol{\alpha_n}). \tag{1}$$

We view these mappings $\boldsymbol{\alpha_i}$ as unary operations on S. As the mapping $S \to S^n$ is one-to-one and onto, it has an inverse $\boldsymbol{\lambda} : S^n \to S$ which we view as an n-ary operation on S.

The operations $\boldsymbol{\alpha_i}$ and $\boldsymbol{\lambda}$ must satisfy the following laws,

$$a\boldsymbol{\alpha_1}...a\boldsymbol{\alpha_n}\boldsymbol{\lambda} = a \tag{2}$$

$$a_1...a_n\boldsymbol{\lambda}\boldsymbol{\alpha_i} = a_i \text{ for } i = 1,...,n \tag{3}$$

for all $a, a_1, ..., a_n \in S$. Conversely, if S is any set on which the operations $\boldsymbol{\alpha_i}$ and $\boldsymbol{\lambda}$ are defined and satisfy (2) and (3), the mapping (1) determines a one-to-one correspondence $S \to S^n$.

Therefore, we introduce the variety $\mathcal{V}_n$ from universal algebra with n unary operations $\boldsymbol{\alpha_1}, ..., \boldsymbol{\alpha_n}$ and one n-ary operation $\boldsymbol{\lambda}$ satisfying the laws (2) and (3). For each pair of integers $n \geq 2$, $r \geq 1$, let $V_{n,r}$ be the free algebra of $\mathcal{V}_n$ with r free generators $\{x_1, ..., x_r\}$.

We can think of the set S associated to the algebraic system $V_{n,r}$ as the set of standard forms defined below.

Remark 1.7. The set $\{x_1, ..., x_r\}$ of free generators will be denoted by $\mathbf{X}$. We write $\langle A \rangle$ to denote the free monoid generated by the elements $\alpha_1, ..., \alpha_n$.

Definition 1.8. The set S of strings in *standard form* is defined as the smallest set of strings over the alphabet $\{x_1, ..., x_r, \alpha_1, \alpha_2, \lambda\}$ satisfying:

- $x_j \in S$ for $j = 1, ..., r$,
- $x_j\Gamma \in S$ for $\Gamma \in \langle A \rangle$,
- if $w_1, w_2 \in S$ and there is no $u \in S$ with $w_i = u\alpha_i$ for $i = 1, 2$ then $w_1w_2\lambda \in S$.

We define unary operations $\boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2$ and a binary operation $\boldsymbol{\lambda}$ on the set S of standard forms by:

- for $\Gamma \in \langle A \rangle$, $(x_j\Gamma)\boldsymbol{\alpha}_i = x_j\Gamma\alpha_i$ for $j = 1, ..., r$,
- for $w_1, w_2 \in S$, if there exists $u \in S$ such that $w_i = u\alpha_i$ for $i = 1, 2$ then $(u\alpha_1, u\alpha_2)\boldsymbol{\lambda} = u$ otherwise $(w_1, w_2)\boldsymbol{\lambda} = w_1w_2\lambda$,
- $((w_1, w_2)\boldsymbol{\lambda})\boldsymbol{\alpha}_i = w_i$.

Therefore, $V_{n,r}$ is isomorphic to the algebraic system $(S, \boldsymbol{\alpha_1}, ..., \boldsymbol{\alpha_n}, \boldsymbol{\lambda})$ satisfying the laws given by (2) and (3).

For a subset X of the algebra $V_{n,r}$, we shall write $X\langle A \rangle$ for the *A-subalgebra* of $V_{n,r}$ generated by elements of X under the application of the operations $\boldsymbol{\alpha}_i$ for $i = 1, ..., n$, and $X\langle \lambda \rangle$ for the *λ-subalgebra* generated by elements of X under the application of the operation $\boldsymbol{\lambda}$.

Lemma 1.9. [3, Lemma 2.3 (i),(ii)] *Suppose that X is a free generating set for $V_{n,r}$ and that $y, y_1, ..., y_n$ are distinct elements of X. Then*

(1)
$$(X \setminus \{y\}) \cup \{y\alpha_1, ..., y\alpha_n\}$$
is also a free generating set. We call this new generating set a single expansion of X; a finite composition of d single expansions is called a d-fold expansion of X.

(2)

$$(X \setminus \{y_1, ..., y_n\}) \cup \{(y_1, ..., y_n)\boldsymbol{\lambda}\}$$

is also a free generating set. This is called a single contraction of X; a finite composition of single contractions is called a contraction.

Higman proved in [3, Lemma 2.3] that any expansion of X freely generates $V_{n,r}$ and that any single contraction of X freely generates $V_{n,r}$, which implies that $\mathbf{X}$ freely generates the algebra $V_{n,r}$. Thus, one can see that $V_{n,r} \cong \mathbf{X}\langle A\rangle\langle\lambda\rangle$.

1.4. **The Higman-Thompson group $G_{n,r}$.** From [3, Section 4], we can define $G_{n,r}$ to be the automorphism group of the free algebra $V_{n,r}$. That is, $G_{n,r}$ is isomorphic to the automorphism group of the algebraic system $(S, \boldsymbol{\alpha_1}, ..., \boldsymbol{\alpha_n}, \boldsymbol{\lambda})$.

Definition 1.10. We define a subset Y of $\mathbf{X}\langle A\rangle$ to be called a *basis* of $\mathbf{X}\langle A\rangle$ if it is a d-fold expansion of $\mathbf{X}$ for some $d \in \mathbb{N}$. We then say that this basis is a finite free basis of $\mathbf{X}\langle A\rangle$ of size d.

Let Y, Z be finite free bases of $\mathbf{X}\langle A\rangle$ of size d. Then any map $\psi : Y \to Z$ induces an *automorphism* of $V_{n,r}$.

Lemma 1.11. [3, Lemma 4.1] *If $\{\psi_1, ..., \psi_k\}$ is a finite subset of $G_{n,r}$ then there is a unique minimal basis Y of $\mathbf{X}\langle A\rangle$ such that $Y\psi_i \subseteq \mathbf{X}\langle A\rangle$, for $i = 1, ..., k$. Any other basis of $\mathbf{X}\langle A\rangle$ with this property is an expansion of Y.*

Therefore, all automorphisms can be realized by a bijective map between bases of the subalgebra $\mathbf{X}\langle A\rangle$.

1.5. **Higman's semi-normal form.** Higman [3, Section 9] constructs a "nicer" finite basis Y (than the one from Lemma 1.11) to study the automorphism ψ of $V_{n,r}$.

For any basis Y, such that $Y\psi \subseteq \mathbf{X}\langle A\rangle$ (which exists by Lemma 1.11), we have elements of Y in one of five different types of orbit under the action of powers of ψ on $\mathbf{X}\langle A\rangle$. For $y \in Y$, the five different types are

1. *Complete infinite orbits.* For any y in such an orbit, $y\psi^i$ belongs to $Y\langle A\rangle$ for all $i \in \mathbb{Z}$, and the elements $y\psi^i$ are all different.
2. *Complete finite orbits.* For any y in such a orbit, $y\psi^n = y$ for some positive integer n, and $y, y\psi, ..., y\psi^{n-1}$ all belong to $Y\langle A\rangle$.
3. *Right semi-infinite orbits.* For some y in the orbit, $y\psi^i$ belongs to $Y\langle A\rangle$ for all $i \geq 0$, but $y\psi^{-1}$ does not. The elements $y\psi^i$, $i \geq 0$, are then, of course, necessarily all different.
4. *Left semi-infinite orbits.* For some y in the orbit, $y\psi^{-i}$ belongs to $Y\langle A\rangle$ for all $i \geq 0$, but $y\psi$ does not. The elements $y\psi^{-i}$, $i \geq 0$, are then, of course, necessarily all different.
5. *Incomplete orbits.* For some y and some non-negative integer n we have $y, y\psi, ..., y\psi^n$ belonging to $Y\langle A\rangle$ but $y\psi^{-1}$ and $y\psi^{n+1}$ do not.

Definition 1.12. [3, Section 9] An element ψ of $G_{n,r}$ is in *semi-normal* form with respect to the basis Y if there are no incomplete orbits *i.e.* no element of Y is in an orbits of type (5).

By [3, Lemma 9.2], for any element ψ of $G_{n,r}$ there exists a basis with respect to which ψ is in semi-normal form. This leads to an examination of the orbits of elements $y\Gamma$ where $y \in Y$ and $\Gamma \in \langle A \rangle$ under the action of powers ψ, where ψ is given in semi-normal form with respect to Y.

Lemma 1.13. [3, Lemma 9.3]
Let $\psi \in G_{n,r}$ be in semi-normal form with respect to the basis Y. Suppose that $y \in Y$, then one of the following holds,
(A) if some $y\Gamma$ is in a complete finite orbit then y itself belongs to a complete finite orbit which consists of elements of Y. In this case we say y is of type (A).
(B) there exists a non-trivial $\Gamma \in \langle A \rangle$ and $n \neq 0$ such that $y\psi^n = y\Gamma$. If $n > 0$ then the orbit containing y is right semi-infinite, if $n < 0$ then the orbit containing y is left semi-infinite. In this case we say y is of type (B).
(C) if $y \in Y$, is not of type (A) nor (B) above, then there exists some $z \in Y$ and non-trivial $\Delta \in \langle A \rangle$ such that $y\psi^i = z\Delta$, where z is of type (B). Then the orbit containing y is infinite. In this case we say y is of type (C).

We will often refer to elements of type (A), (B) and (C). We now introduce a modified definition from [3, Section 9].

Definition 1.14. Let $u \in V_{n,r}$, $\Gamma \in \langle A \rangle$ and $m \in \mathbb{Z}$. We call u a *characteristic element* of ψ^m and Γ a *characteristic multiplier* if $u\psi^m = u\Gamma$. The element and the multiplier are proper if Γ is non-trivial.

We note by [3, Theorem 9.4] that an automorphism ψ (given in semi-normal form with respect to Y) is of infinite order if ψ^m ($m \neq 0$) has a proper characteristic element *i.e.* $y\psi^m = y\Gamma$ for $y \in Y$ and non-trivial $\Gamma \in \langle A \rangle$.

A stronger definition than semi-normal form was introduced in [3, Section 9].

Definition 1.15. An element ψ of $G_{n,r}$ is in *quasi-normal* form with respect to the basis Y if it is in semi-normal form with respect to Y, but not with respect to any proper contraction of Y.

For all $\psi \in G_{n,r}$ we can do the following: put ψ in quasi-normal form with respect to a basis Y, for any $u, v \in V_{n,r}$ we can determine if u, v are in the same orbit of ψ and determine which integers m give $v = u\psi^m$ if they are in the same orbit [3, Lemma 9.7]; and if $m > 0$ we have $u\psi^i \in Y\langle A \rangle$ for all $i = 1, ..., m-1$ [3, Lemma 9.6].

2. Conjugacy Problems in the Higman-Thompson group $G_{n,r}$

We will first recall some more relevant information from [3, Section 9] for the readers connivence.

2.1. **Higman's ψ-admissible subalgebras V_P and V_{RI}.** Given $\psi \in G_{n,r}$, we define $V_{P,\psi}$ to be the set of $v \in V_{n,r}$ such that v is in a finite orbit under the action of ψ and $V_{RI,\psi}$ to be the set of $v \in V_{n,r}$ such that v is in an infinite or semi-infinite orbit under the action of ψ. Where there is no ambiguity, we will write V_P for $V_{P,\psi}$ and V_{RI} for $V_{RI,\psi}$.

From [3, Theorem 9.5] (which we state below) we see that $V_{P,\psi}$ and $V_{RI,\psi}$ are subalgebras invariant under the action of the automorphism ψ and $V_{n,r}$ is a free product of the subalgebras $V_{P,\psi}$ and $V_{RI,\psi}$. That is, $V_{n,r}$ can be seen as the coproduct of the sub-algebras V_{RI} and V_P in the category of free algebras We will refer to V_P as the periodic subalgebra and V_{RI} as the regular infinite subalgebra.

Theorem 2.1. [3, Theorem 9.5] *The finitely generated free algebra $V_{n,r}$ is a free product of the ψ-admissible subalgebras V_P and V_{RI}. If $\psi_P = \psi|_{V_P}$ and $\psi_{RI} = \psi|_{V_{RI}}$ then for two automorphisms ψ and φ, ψ is conjugated to φ by $g \in G_{n,r}$ if and only if ψ_P is conjugated to φ_P by a map $g^{(P)}$ and ψ_{RI} is conjugate to φ_{RI} by a map $g^{(RI)}$, where g is given by the composition of maps $g^{(P)}, g^{(RI)}$.*

This Theorem allows us to consider the torsion and regular infinite part of an element of $G_{n,r}$ separately. Hence, we can consider separately the elements that conjugate the torsion part and the regular infinite part of an element.

Definition 2.2. Let ψ be in quasi-normal form with respect to a finite basis Y. Then ψ is a *regular infinite* element of $G_{n,r}$ with respect to Y if there exists no $y \in Y$ in a finite orbit.

Definition 2.3. Let ψ be an element of $G_{n,r}$ in quasi-normal form with respect to the basis Y. Then ψ is a *periodic element* of $G_{n,r}$ if for all $y \in Y$, y is in a complete finite orbit.

2.2. **A condition for conjugacy.** From [3, Section 9], we have a solution to the conjugacy problem for the groups $G_{n,r}$, for $n \geq 2$ and $r \geq 1$. Below we summarize part of the proof of the following theorem.

Theorem 2.4. [3, part of Theorem 9.3] *The conjugacy problem for the group $G_{n,r}$ is solvable.*

By Theorem 2.1, we can just consider the torsion elements and regular elements of $G_{n,r}$.

Definition 2.5. Let ψ be a torsion element of $G_{n,r}$ in semi-normal form with respect to the basis Y. We define the cycle type of ψ to be the sequence of lengths of finite orbits on Y under the action of ψ.

The following is a consequence of [3, Section 6].

Corollary 2.6. *Let ψ and φ be torsion elements of $G_{n,r}$ in semi-normal form with respect to the bases X and Y. Then, ψ is conjugate to φ if and only if ψ and φ have the same cycle type.*

For regular infinite elements we take X,Y bases such that ψ, φ are in quasi-normal form with respect to them.

Following [3, Section 9], an equivalence relation is introduced on the elements of X (and thus on the elements Y respectively).

Definition 2.7. Let ψ be a regular infinite element in quasi-normal form with respect to X. The equivalence relation on the elements of X, $\equiv$, is defined to be the least equivalence relation such that $x \equiv x'$ whenever some $x\Gamma$ and $x'\Delta$ are in the same orbit of ψ, for $\Gamma, \Delta \in \langle A \rangle$.

For each equivalence class $\mathcal{X}_j$, an element of type (B) $x \in \mathcal{X}_j$ is picked. Then x is a proper characteristic element (by Definition 1.14) of a proper power m_x of ψ with some multiplier Γ_x *i.e.* $x\psi^{m_x} = x\Gamma_x$. If an isomorphism exists, then $x\rho$ must be a characteristic element of φ^{m_x} with multiplier Γ_x.

If no characteristic elements with the "right" multiplier exists, then ρ does not exist.

Definition 2.8. Let ψ be a regular infinite element of $G_{n,r}$ in quasi-normal form with respect to X. For all elements $x \in X$ of type (B), we find the corresponding $m_x \in \mathbb{Z}$ and $\Gamma_x \in \langle A \rangle$ (characteristic multiplier) such that $x\psi^{m_x} = x\Gamma_x$.

The set

$$\mathcal{M}_\psi = \{(m_x, \Gamma_x) | x \in X, \text{ is an element of type (B))}\}$$

of pairs is called the set of characteristic multipliers and powers for ψ.

Lemma 2.9. *Suppose that ψ and φ are conjugate regular infinite elements of $G_{n,r}$. Then the set of characteristic multipliers and powers $\mathcal{M}_\psi$, $\mathcal{M}_\varphi$ for ψ and φ coincide.*

Proof. Let X, Y be bases such that ψ, φ are in semi-normal form with respect to them. For the basis X we can look at the equivalence relation, as in Definition 2.7, on the elements for this basis.

For each equivalence class $\mathcal{X}_j$ we pick an element x in $\mathcal{X}_j$ of type (B) (see Lemma 1.13). Then, x is a proper characteristic element by Definition 1.14 of a proper power m_x of ψ with some multiplier Γ_x *i.e.* $x\psi^{m_x} = x\Gamma_x$. If an isomorphism ρ exists (*i.e.* ψ is conjugated to φ by ρ) then $x\rho$ will be a characteristic element of φ^{m_x} with multiplier Γ_x. Thus, $x\rho$ must, by Lemma 1.13, belong to a semi-infinite orbit of φ. (If no orbit of appropriate characteristic exists, ρ did not exist.)

Therefore, we can look at each semi-infinite orbit and see whether or not its elements are characteristic elements with the "right" multiplier (if one is, they all are by the definition of semi-infinite orbit).

Hence, the set of characteristic multipliers for ψ and φ coincide. □

The above lemma gives a necessary condition for conjugacy (not a sufficient one). However, once we have satisfied Lemma 2.9, the algorithm Graham Higman constructed in [3, Section 9] to prove Theorem 2.4 gives rise to a finite set of candidate conjugators to check.

2.3. **Power conjugacy problem for the group** $G_{n,r}$**.** We will now prove Theorem 1.2. We can break this problem down into two subsections by Theorem 2.1: when ψ, φ are torsion elements and when ψ, φ are regular infinite elements.

2.3.1. *Torsion elements.* Let X, Y be bases such that torsion elements ψ, φ are in quasi-normal form with respect to them.

As ψ and φ are finite, there are only a finite number of choices for the pair $(a, b) \in \mathbb{Z}^2$ such that $|a| \neq |b|$. We can therefore take all pairs and apply the conjugacy problem criteria for torsion elements.

2.3.2. *Regular infinite elements.* Let X, Y be bases such that the regular infinite elements ψ, φ are in quasi-normal form with respect to them. If we want to find integers a, b with $|a| \neq |b|$ such that ψ^a is conjugate to φ^b, we need to examine all elements of X (resp. Y) of type (B).

Each element $x \in X$ of type (B) is a proper characteristic element of a proper power m_x of ψ with some multiplier Γ_x *i.e.* $x\psi^{m_x} = x\Gamma_x$. Similarly, for the elements $y \in Y$ of type (B) are proper characteristic element of a proper power m_y of ψ with some multiplier Γ_x *i.e.* $y\varphi^{m_y} = x\Delta_y$. Both m_x and m_y are taken to be the smallest possible integers.

A condition for conjugacy is that the set of characteristic multipliers and powers $\mathcal{M}_\psi = \{(m_x, \Gamma_x)\}_{x \in X}$ and $\mathcal{M}_\varphi = \{(m_y, \Delta_y)\}_{y \in Y}$ necessarily coincide. Therefore, as $x\psi^{am_x} = x(\Gamma_x)^a$ and $y\varphi^{bm_y} = x(\Gamma_y)^b$, we need to be able to pair up (m_x, Γ_x) and (m_y, Δ_y) such that

$$a_i m_x = b_j m_y \text{ and } \Gamma_x^{a_i} = \Delta_x^{b_j},$$

for $a_i, b_j \in \mathbb{Z} \setminus \{0\}$.

However, there are only a finite number of possibilities for a_i, b_j, if we require that $\gcd(a_i, b_j) = 1$ (given that the sets $\{(m_x, \Gamma_x)\}_{x \in X}$ and $\{(m_y, \Delta_y)\}_{y \in Y}$ are finite). We can therefore take all a_i which are coprime, call them $a_i^{'}$, and all b_j which are coprime, call them $b_j^{'}$, then form the products $a = \prod_i a_i^{'}$, $b = \prod_j b_j^{'}$.

There will be a finite set of pairs of elements (a, b) satisfying the above condition. We then apply the conjugacy problem solution, Theorem 2.4, to ψ^a and φ^b. If ψ^a is conjugate to φ^b, then Theorem 2.4 constructs a conjugator.

The proof of Theorem 1.2 now follows from the above.

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